

# **Nonlinear Dynamics, the Missing Fundamental, and Harmony**

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Nonlinear dynamics offers a theoretical basis for the functioning of our sensory systems, and in particular hearing. Auditory sensitivity, otoacoustic emissions, harmonic generation, combination tones and the perception of pitch are some of the nonlinear effects that have been observed in psychoacoustics and modelled using nonlinear techniques. In particular, the perception of pitch is fundamental for music perception and has been shown to be intimately related to nonlinear phenomena. The missing fundamental problem (or residue perception) is related to the perception of frequencies which are not actually present in the stimulus, a typical signature of nonlinear phenomena. It has been demonstrated that the missing fundamental problem can be accurately described by the behaviour of a kind of dynamical attractors called three-frequency resonances. This fact is pertinent to the interpretation of the auditory system at at least two different levels of description: the physiology of neural centres, and musical perception. The explanation of pitch perception on the basis of dynamical attractors suggests the existence of corresponding real frequencies at the physiological level and thus may contribute to explain the organization, functionality, and information flow related to auditory neural ganglia. As a related consequence, tonal perception, which has long been associated with the missing fundamental and related psychoacoustical phenomena can be thought of as a resonance interaction at the level of the dynamics of the neural system. A simple but fundamental implication is that the existence of harmonic intervals, and more generally the existence of harmonic chords, can be explained in terms of the stability of dynamical attractors. In this framework, dissonance can be ascribed to a loss of stability of a dynamical attractor ending in a more stable one depending on the corresponding dynamical basins of attraction. In musical terms, the transition from an unstable attractor (dissonance) to a more stable one (consonance) corresponds to the concept of dissonance resolution.